June 6, 1901.

A List of the Presents received was laid on the table, and thanks ordered for them.

The following Papers were read :—

I. "On the Electric Response of Inorganic Substances. Preliminary Notice." By Professor J. C. BOSE. Communicated by Sir M. FOSTER, Sec. R.S.

II. "On Skin Currents. Part I.—The Frog's Skin." By Dr. A. D. WALLER, F.R.S.

III. "Vibrations of Rifle Barrels." By A. MALLOCK. Communicated by LORD RAYLEIGH, F.R.S.

IV. "The Measurement of Magnetic Hysteresis." By G. F. C. SEARLE and T. G. BEDFORD. Communicated by Professor J. J. THOMSON, F.R.S.

V. "A Conjugating 'Yeast.'" By B. T. P. BARKER. Communicated by Professor MARSHALL WARD, F.R.S.

VI. "Thermal Adjustment and Respiratory Exchange in Monotremes and Marsupials: a Study in the Development of Homo-thermism." By Professor C. J. MARTIN. Communicated by E. H. STARLING, F.R.S.

VII. "On the Elastic Equilibrium of Circular Cylinders under certain Practical Systems of Load." By L. N. G. FILON. Communicated by Professor EWING, F.R.S.

VIII. "The Measurement of Ionic Velocities in Aqueous Solution, and the Existence of Complex Ions." By B. D. STEELE. Communicated by Professor RAMSAY, F.R.S.

"Vibrations of Rifle Barrels."* By A. MALLOCK. Communicated by LORD RAYLEIGH, F.R.S. Received May 2,—Read June 6, 1901.

It has long been known that a shot fired from a rifle does not in general start from the muzzle in the direction occupied by the axis of the barrel at the first moment of ignition of the charge.

* The greater part of the notes from which this paper is drawn were made in 1898, but since that time the interesting experiments of Messrs. Cranz and Koch, of Stuttgart, on the same subject have been published, and I have looked through my notes again and put them in their present form, as it may be of some interest to compare results obtained in such different ways.

The late W. E. Metford was, I believe, the first to point out the origin of this deviation, showing by experiment that it was due to the unsymmetrical position which the mass of the stock held as regards the barrel; and, further, that if the initial direction of the shot passed below the apparent direction of aim when the rifle was held in the ordinary position, the initial direction would be high if the rifle were aimed upside down, and to the right or left if the plane of the stock were horizontal and the stock itself to the left or right of the barrel.

He showed, in fact, that the initial direction of a shot lay on a cone, whose axis was the axis of the barrel at the instant before the ignition of the powder, and in a plane containing the axis of the barrel and the centre of gravity of the rifle, and he rightly attributed the deviation of the shot to the bending couple acting on the barrel, due to the direction of the force causing the recoil not passing through the centre of gravity of the rifle.

The object of this paper is to examine this problem of "flip" or "jump," as it is called, from a mathematical point of view, and to show what effect may be expected from given variations either in the length of the barrel, the nature of its attachment to the stock, or the nature of the explosive employed.

The investigation is not merely a matter of curiosity, but has an important bearing on the accuracy of rifle shooting, and until some method is introduced, not of avoiding "jump," but of suitably regulating its variation with the variation of explosive force, I think no great advance will be made on the precision already attained in modern rifles.

This precision is already considerable, and, roughly speaking, any good modern rifle will shoot with a probable deviation of considerably less than 2′ from the intended path. When the results indicated in the course of this paper are considered, it seems wonderful that such accuracy should be possible, and it speaks well for the quality and uniformity of the ammunition that such good shooting should be common.

The problem of "jump" may be stated mathematically thus:—"An elastic tube, to which a mass is unsymmetrically attached, is subjected for a given time to a couple of arbitrary magnitude. Determine the subsequent motion." To solve this problem we must consider the tube and its attached mass as forming a single system, and examine what are the natural modes of vibration of this system, and what their natural periods. The arbitrary couple must be expressed in an harmonic series as a function of time, and the forced vibration which each term of this series will evoke in the system calculated.

To represent the initial conditions (namely, that at the moment before the explosion the barrel is at rest and unrestrained), such free

vibrations of the system must be supposed to exist as, in combination with the forced vibration, will satisfy these conditions. The subsequent motion will then be determined by taking the sum of the forced and free vibrations as long as the arbitrary couple acts, and when this has ceased to act, the sum of the free vibrations only.

If the system could be represented by a uniform rod, the solution might at once be expressed in symbols, since the theory of the transverse vibrations of rods and tubes is well known. When we come, however, to a "system" like a rifle, although in many respects its behaviour may be compared with that of a uniform elastic rod of "equivalent length," the ratio between the periods of the vibrations of its various modes are altered, and recourse must be had to experiment to determine both the natural periods and the position of the nodes.

As far, however, as the rifle can be considered as being represented by an equivalent rod, it must be looked upon as being free at both ends at the moment of firing, because the motion communicated to the rifle is so small at the time the shot leaves the muzzle, that the constraint which hands and shoulders can impose on it is negligible compared to the acceleration forces called into play by the explosion.

This being so, the slowest vibration of which the system is capable is that with two nodes. The next in order of rapidity will have three nodes, and so on, as shown in the figures 1, 2, 3.

Fig. 1.—Mode I.

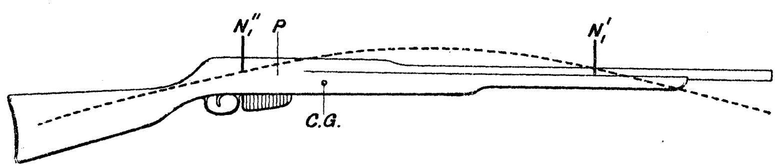


Fig. 2.—Mode II.

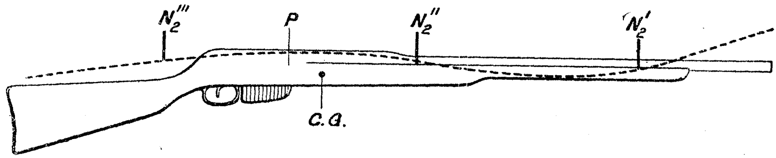
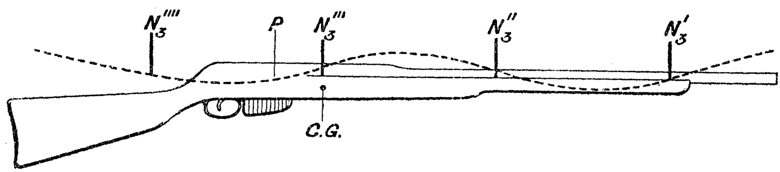


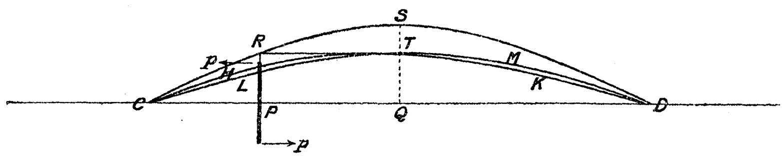
Fig. 3.—Mode III.



The figure assumed by the muzzle end of the barrel will be nearly exactly the same in each mode as the figure assumed in the corresponding mode by an uniform rod whose length is such as to make the distance of the node from its free end equal to the distance from the node to the muzzle of the rifle.

The couple which acts on the barrel during the explosion is measured by the rate at which the shot is accelerated, the distance of the axis of the barrel from the centre of gravity of the rifle. The effect of a given couple in causing a particular mode of vibration in the barrel depends on its point of application with reference to the nodes of the system as well as on its magnitude.

FIG. 4.



$QS = y_{QQ}$. $PR = y_{PQ} = QT = y_{QP}$.

CHTKD is the curve into which CD is bent by F acting at P.

CLTMD is that part of the deformation which belongs to the mode of vibration which has nodes at C and D.

If in fig. 4, C and D are two adjacent nodes belonging to some particular mode of vibration, it is evident that a couple applied midway between C and D would not cause any displacement of the system in this mode.

If a is the distance between the nodes C D and a couple pd at point P distant x from c, there will be

(1) A downward force at C $= pd/2x$ with an equal upward force at P, and

(2) An upward force at D $= \dfrac{pd}{2(a-x)}$ with an equal downward force at P.

On the whole, therefore, there is at P an upward force acting

$$= \frac{pd}{2}\left(\frac{1}{x} - \frac{1}{a-x}\right), \qquad \text{or} \qquad F = \frac{pd}{2}\left(\frac{a-2x}{x(a-x)}\right).$$

Suppose $y_{QQ} = cF$ to be the displacement which the force F would cause if acting at the point Q, midway between C and D. It is known that if a force F acting at Q causes a displacement y_{PQ} at P, the same force acting at P will cause a displacement y_{PQ} at Q, that is $y_{PQ} = y_{QP}$.*

* This theorem is due to Lord Rayleigh.

Approximately, the equation to the curve between the nodes C and D for the mode of vibration which has these nodes may be taken as a simple harmonic function of x

or $$y = \zeta_{QQ} \sin 2\pi \frac{x}{a};$$

hence the displacement at P due to F acting at Q, and the displacement at Q due to F acting at P, are each equal to

$$\mathrm{CF} \sin 2\pi \frac{x}{a},$$

or $$y_{QP} = c \frac{pd}{2} \frac{a-2x}{x(a-x)} \sin 2\pi \frac{x}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1).$$

In a rifle the point of application of the couple is settled by the nature of the connection between the stock and the barrel, and it is a matter of great difficulty to make certain how the strains are distributed. The actual maximum pressure in the barrel which is spoken of as "chamber pressure" is known for various small arms and various explosives with considerable accuracy; but the curve of pressure in terms of the travel of the shot along the barrel is much more difficult to ascertain. In this paper, therefore, I shall consider several types of such curves in order to show what effects are to be looked for as the pressure curve changes its character.

The condition fulfilled in each of the pressure curves considered is that each must give the same muzzle velocity to the shot by acting on it through the length of the barrel, and in the numerical results given the velocity and weight of the projectile are taken as 2000 feet per second and 215 grains respectively, with an effective length of barrel of 2·3 feet, these being nearly the velocity, weight, and length of barrel used in the Lee-Enfield rifle.

The simplest case of all (and the furthest removed from truth) is that of a uniform pressure acting on the base of the shot throughout the length of the barrel.

Here we have, if p_0 is the acceleration, v_m the muzzle velocity, $\mathfrak{T}$ the time taken by the shot in reaching the muzzle, and l the length of the barrel,

$$v_m = p_0 \mathfrak{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2),$$

$$l = p_0 \frac{\mathfrak{T}^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3),$$

$$p_0 = \frac{v_m^2}{2l} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4),$$

$$\mathfrak{T} = \frac{2l}{v_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5).$$

Putting $v = 2000$ f.s., and $l = 2{\cdot}3$ ft.,

we have $p_0 = 860{,}000$ f.s.s., $\mathfrak{T} = 0{\cdot}0023$ secs.

An acceleration of 860,000 is about $27{,}000g$, so that a uniform force of 27,000 times its own weight, or 835 lbs., would give the 215-grain shot its observed velocity in the actual length of the barrel.

With a uniform force, the pressure curve in terms of space is the same, of course, as if expressed in terms of time; but for any other case we must, for the purpose of this paper, express the pressure curve (which experiment would give in terms of the distance travelled by the shot in the barrel) in terms of time.

The pressure at time t being p, we have

$$p = \frac{dv}{dt}; \quad \text{also} \quad \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}, \quad \text{and} \quad p\,ds = v\,dv$$

$$\therefore \quad v = \sqrt{(2\textstyle\int p\,ds)} \qquad (6);$$

and

$$t = \int \frac{ds}{\sqrt{(2\int p\,ds)}} \qquad (7).$$

If we take the case of the pressure decreasing uniformly with the travel of the shot, it is easy to show by (5) and (6) (although the analogy with the force acting on a pendulum or spring at once suggests it), that the velocity and position of the shot are:—

$$s = l\left(1 - \cos t\sqrt{\frac{p_0}{l}}\right) \qquad (8),$$

$$v = \sqrt{p_0 l}\sin t\sqrt{\frac{p_0}{l}} \qquad (9),$$

$$p_0 = \frac{v_m^2}{l} \qquad (10),$$

$$\mathfrak{T} = \frac{\pi}{4}\frac{l}{v_m} \qquad (11).$$

With the before-mentioned values for l, v, and w, $p_0 = 1{\cdot}174 \times 10^6$ f.s.s. and $\mathfrak{T} = 0{\cdot}00171$ second.

One more case by way of example will suffice. Let the pressure decrease uniformly with the time so that

$$p = p_0\left(1 - \frac{t}{\mathfrak{T}}\right) \qquad (12).$$

From this we get

$$v = p_0 t - p_0\frac{t^2}{2\mathfrak{T}} \qquad (13),$$

$$s = p_0\frac{t^2}{2}\left(1 - \frac{t}{3\mathfrak{T}}\right) \qquad (14),$$

and the relation between p and s is

$$s = \frac{l}{2}\left(2 - 5\frac{p}{p_0} + 4\frac{p^2}{p_0{}^2} - \frac{p^3}{p_0{}^3}\right) \ldots\ldots\ldots\ldots \quad (15).$$

From (13) (14), using the above values for v_m and l,

$$p_0 = 2{\cdot}32 \times 10^6 \text{ f.s.s.} \qquad \mathfrak{T} = 0{\cdot}00173 \text{ sec.}$$

The three cases are illustrated in diagrams 5, 6, 7, in which the various curves show the pressure, velocity, and time elapsed since the beginning of the motion during the passage of the shot through the barrel.

Diagrams 8, 9, 10 show the pressure in terms of time, and it is these curves which have to be represented by a harmonic series.

In order to avoid having a constant term at the beginning of the series, the fundamental t is taken equal to $2\mathfrak{T}$.

Then by the ordinary rules for finding the coefficient of a Fourier series, the succession of "battlements" which form the pressure curve in case 1 (uniform acceleration), we find

$$p = p_0 \frac{4}{\pi}\left\{\sin 2\pi\frac{t}{t_1} + \frac{1}{3}\sin 3\left(2\pi\frac{t}{t_1}\right) + \frac{1}{5}\sin 5{\cdot}2\pi\frac{t}{t_1} + \&c.\right\} \quad (16).$$

In case 2, where the pressure curve is a succession of half-lengths of a simple harmonic curve, the general coefficient of the nth term is

$$p_0 \frac{2}{\pi}\frac{4n}{4n^2 - 1},$$

and the series is

$$p = p_0 \frac{2}{\pi}\left\{\frac{4}{3}\sin 2\pi\frac{t}{t_1} + \frac{8}{15}\sin 2\left(2\pi\frac{t}{t_1}\right) + \&c.\right\} \ldots\ldots \quad (17).$$

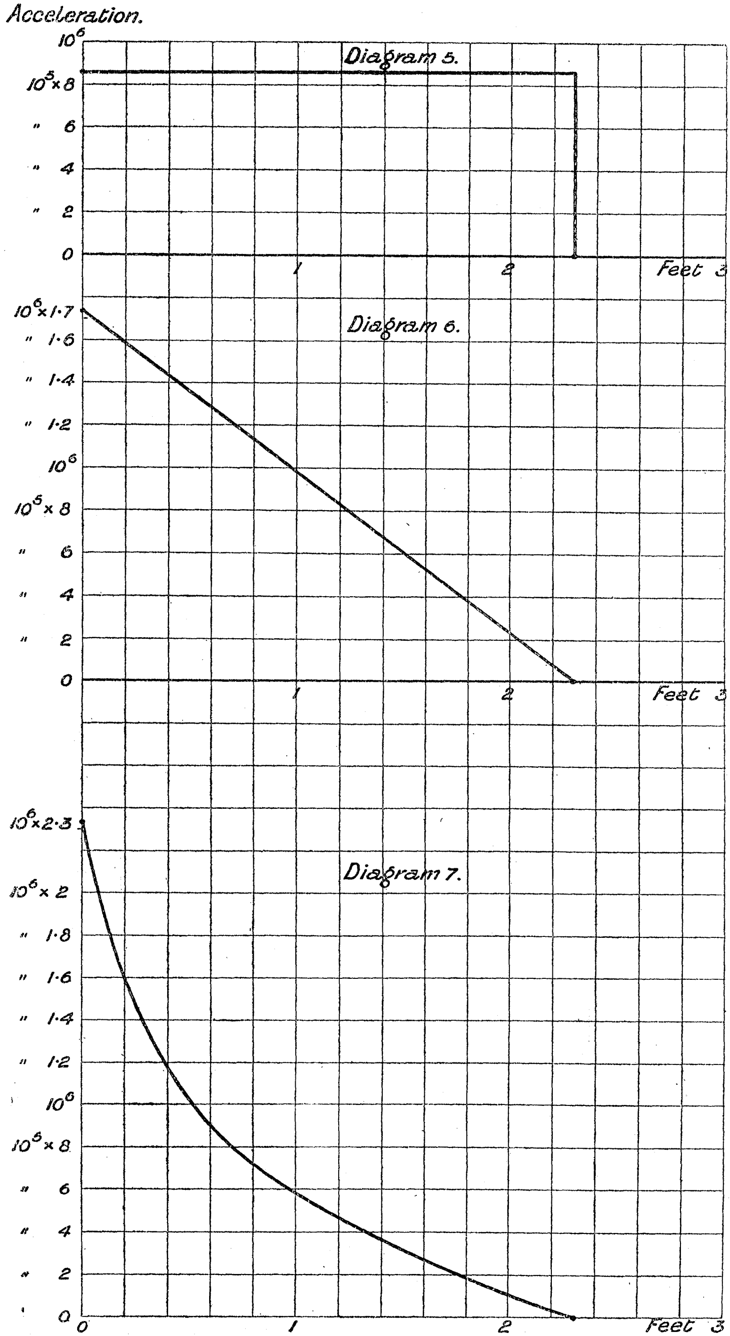
The series for case (3), where $p = p_0\left(1 - \frac{t}{\mathfrak{T}}\right)$, is

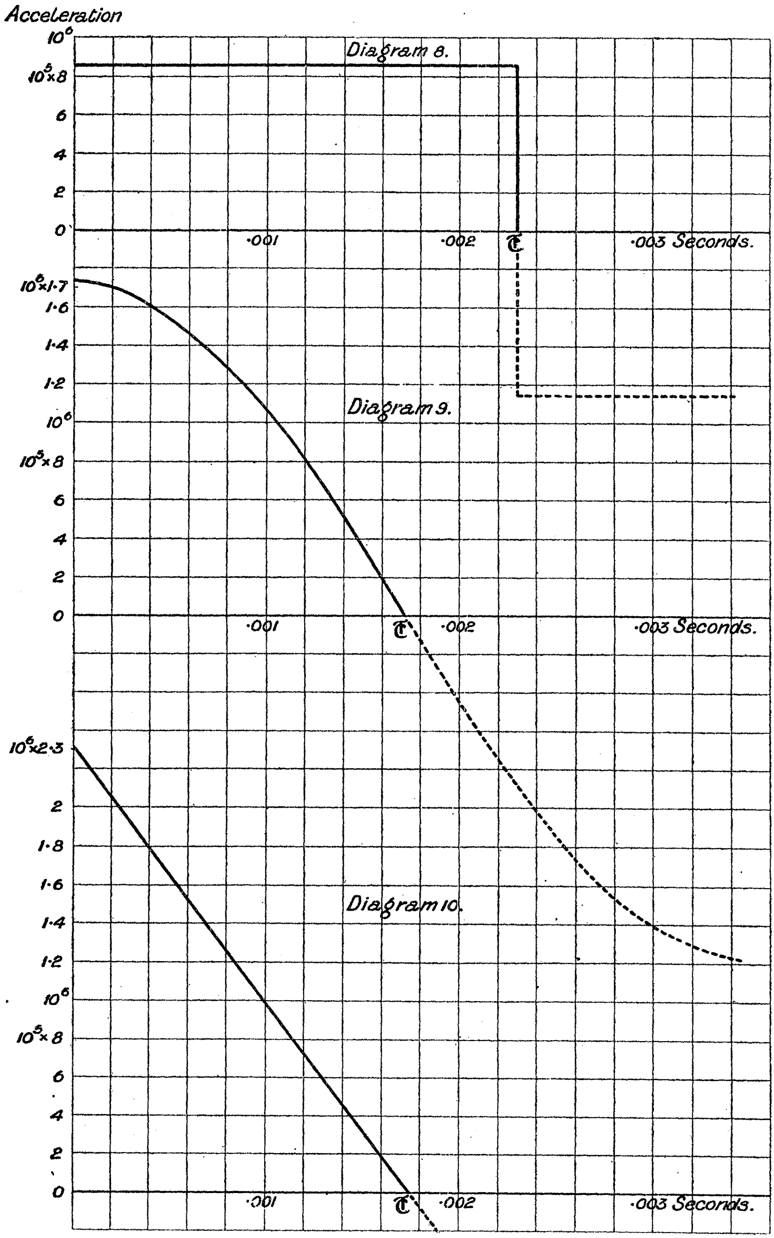
$$p = p_0 \frac{2}{\pi}\left\{\sin 2\pi\frac{t}{t_1} + \frac{1}{2}\sin 2\left(2\pi\frac{t}{t_1}\right) + \frac{1}{3}\sin 3\left(2\pi\frac{t}{t_1}\right) + \&c.\right\} \quad (18).$$

The coefficients in series 17 and 18 soon become sensibly equal in the corresponding higher terms of each.

In the cases just considered, except the first, it is assumed that the pressure at the muzzle is zero, which of course is not true, but the existence of a terminal pressure can be readily represented by adding a series of the form of (16) of suitable magnitude. The effect of this is to increase the relative importance of the first and all the odd terms.

We must now examine the forced vibrations which each term of the series expressing the accelerating pressure would set up in the rifle,


Acceleration.
Diagram 5.
10^6
$10^5 \times 8$
" 6
" 4
" 2
0
1
2
Feet 3
Diagram 6.
$10^6 \times 1{\cdot}7$
" 1·6
" 1·4
" 1·2
10^6
$10^5 \times 8$
" 6
" 4
" 2
0
1
2
Feet 3
Diagram 7.
$10^6 \times 2{\cdot}3$
$10^6 \times 2$
" 1·8
" 1·6
" 1·4
" 1·2
10^6
$10^5 \times 8$
" 6
" 4
" 2
0
0
1
2
Feet 3



supposing that the harmonic couple it represents continued to act. If $T_1, T_2 \ldots T_m$ are the natural periods of the various modes in which the rifle can vibrate, and d the distance of the centre of gravity from

the axis of the barrel, the forced oscillation which the nth term in the series will evoke in the mth mode of the rifle will be, when expressed as the angle through which some particular part of the system bends during the oscillation, is

$$\theta = \theta_m p d \mathrm{A}_n \frac{1}{1-q^2_{nm}} \sin 2n\pi \frac{t}{t_1} \quad \text{..............} \quad (19).$$

In this expression θ_m is the angle at the place of observation which the unit couple would cause if acting to produce a displacement of the system in the mth mode (the values of θ_m can be found approximately by statical experiments on bending).

A_n is the numerical coefficient of the nth term of the harmonic series, and

$$q_{nm} = \frac{\mathrm{T}_m}{t_n} \quad \text{or} \quad \frac{\mathrm{T}_m}{nt_1} \quad \text{....................} \quad (20).$$

To represent the initial conditions, which are that the moment before the explosion the barrel is at rest and unstrained, it suffices to suppose the co-existence of free oscillations of the system, with phases and amplitudes such as to make the velocity and displacement zero when $t = 0$. If a and b are the amplitudes of the forced and free vibrations respectively, we have

$$a \sin 2\pi \frac{t}{t_1} + b \sin 2\pi \frac{t}{\mathrm{T}_n} = 0 \quad \text{................} \quad (21),$$

and
$$\frac{2\pi}{t_n} a \cos 2\pi \frac{t}{t_n} + \frac{2\pi}{\mathrm{T}_m} b \cos 2\pi \frac{t}{t_m} = 0 \quad \text{...........} \quad (22),$$

whence
$$q_{nm} = -\frac{b}{a} \quad \text{.........................} \quad (23),$$

hence the free vibration, which at $t = 0$ leaves the system at rest, so far as the oscillation excited by the nth term in the mth mode is concerned, has q_{nm} times the amplitude of the corresponding forced vibration.*

It is convenient in the complete expression for displacement to refer to the natural periods of the system, which are constant, rather than to the periods contained in the pressure curve. So, substituting for t_n its value T_m/q_{nm}, we have for the angular displacement of the system at that time after the explosion (*i.e.*, for the sum of the forced and free vibrations at that time due to the term and mode under consideration)

* For the purposes of this paper it is not necessary to consider the gradual extinction of the free vibrations, for the number of periods involved is so small, even for the highest component taken into account, that extinction will not materially affect the amplitude.

$$\theta_{nm} = \theta_m p d \mathrm{A}_n \frac{1}{1-q^2_{nm}} \left(q_{nm} \sin 2\, p \frac{t}{\mathrm{T}_m} - \sin q_{nm} 2\pi \frac{t}{\mathrm{T}_m} \right) \dots \quad (24).$$

Diagram 11 shows the curves represented by the function

$$\frac{q \sin \phi \; - \; \sin q\phi}{1-q^2} \text{ from } \phi = 0 \text{ to } \phi = 2\pi \text{ and } q = 0{\cdot}6 \text{ to } q = 4.$$

When $q = 1$ this expression takes the form of $\propto . 0$ which, evaluated in the usual way, gives

$$\frac{\phi \cos \phi \; - \; \sin \phi}{2}.$$

I will now apply the above results to examine the form of the Lee-Enfield rifle at the moment the shot leaves the barrel, assuming that the pressure developed during the explosion is that shown in fig. 10, taking into consideration the first three terms of the harmonic series for that curve and the first three modes of vibration of the rifle.

For this rifle it was found by experiment* that a couple of 1 foot-lb. acting at the nodes caused at the muzzle the following deflections :—

Mode I	....	$\Theta_1 = 1'{\cdot}13$
Mode II	....	$\Theta_2 = 0'{\cdot}765$
Mode III	....	$\Theta_3 = 0'{\cdot}565$

In the authorised 'Text-book for Military Small Arms' the initial pressure in the chamber of the Lee-Enfield is given as 15 tons per square-inch.

The area of the base of the shot is 0·0725 square-inch, so that the initial pressure on the shot is 1·09 tons or 2450 lbs. Since the weight of the shot itself is 215 grs., the force acting on it is $\frac{7000}{215} \times 2450$, nearly 80,000 times its own weight. Multiplying this by g, the acceleration which the shot would undergo in the absence of friction in the barrel is 2,560,000 feet per second per second.

In case 3 (14) the initial pressure was found to be 2,320,000 feet per second per second, so that, allowing for the force required to press the shot into the rifling and the friction in the barrel, it seems probable that the pressure curve of case 3 represents with some degree of approximation the actual acceleration which the shot experiences.

* It would occupy too much space to describe these experiments in detail. They were made by loads suitably placed on the rifle, and the deflections caused by them were measured by optical means. The deflections so found were reduced to what they would have been had the action of the couples been concentrated at the nodes. In virtue of the approximate straightness of the free end of a vibrating rod, the angular deflection at the muzzle was taken as equal to the angular deflection at the nearest node. Hence the deflections above given are rather less than the true values.

The centre of gravity of the rifle is just an inch below the axis of the barrel, and, taking the accelerative pressure on the shot as 2250 lbs., the bending couple at the first instant is 187 ft.-lbs.

Also $A_1 = \frac{2}{\pi}, \quad A_2 = \frac{2}{2\pi}, \quad A_3 = \frac{2}{3\pi}.$

Thus

$p_0 d A_1 = 118$ ft.-lbs., $p_0 d A_2 = 59$ ft.-lbs., $p_0 d A_3 = 40$ ft.-lbs.,

Table I.

$\theta_1 p_0 d A_1 = 133'$	$\theta_2 p_0 d A_1 = 90'{\cdot}5$	$\theta_3 p_0 d A_1 = 66'{\cdot}5$
$\theta_1 p_0 d A_2 = 66'{\cdot}5$	$\theta_2 p_0 d A_2 = 45'$	$\theta_3 p_0 d A_2 = 33'{\cdot}3$
$\theta_1 p_0 d A_3 = 45'$	$\theta_2 p_0 d A_3 = 30'$	$\theta_3 p_0 d A_3 = 22'{\cdot}2$

These are the angular displacements which the muzzle would undergo if in each case it experienced the full statical effect of couple corresponding to the first, second, and third term of the series representing the explosion curve acting so as to deform the system in the first, second, or third mode.

Owing, however, to the position of the point of application of the couples with reference to the nodes of the various modes (see I, and figs. 2 and 3), it appears that for the first mode the couple will cause 0·88 of its full effect, as for this mode the node N_1'' coincides nearly with the point of application of the couple. The nodes N_2' and N_2'' of the second mode fall at such a distance from P as to reduce the effect of the couples to about 0·35 of the above value. And the reduction is about 0·6 for displacements in the third mode.

The following table is an approximation to the actual values of—

Table II.

	m.		
n.	1.	2.	3.
1	117′	31′·5	39′
2	59′·5	15′·8	20′
3	39′·5	10′·5	13′·2

To determine the periods T_1, T_2, T_3, namely the natural periods of the rifle in the first, second, and third modes, experiments were made by tapping the barrel so as to excite the modes in question, and determining the notes emitted by comparison with tuning forks. The

positions of the nodes were found by noting the position of the points of support which did not damp the vibrations in each mode examined.

The results were as follows :—

Table III.

	Frequency.	Period.	Distance of nearest node from muzzle.
	Per sec.	sec.	in.
Mode I	66	0·015	12·5
Mode II	172	0·00575	8·6
Mode III	395	0·00253	6·5

In case 3, again, the value found for $\mathfrak{T}$ was 0·00173 second, hence for the assumed ammunition $t_1 = 0{\cdot}00346$ second.

We can now construct a table of the value of q_{nm}.

Table IV.

Values of q_{nm} for $m = 1$ to $m = 3$, $n = 1$ to $n = 3$.

	T_1.	T_2.	T_3.
t_1	4·3	1·64	0·72
t_2	8·6	3·28	1·44
t_3	17·2	4·92	2·94

The abscissa on Diagram 11, which corresponds to the time $\mathfrak{T}$ will be

$$\text{For Mode I} \ \ldots\ldots\ldots \quad 2\pi\frac{\mathfrak{T}}{T_1} = 42^\circ{\cdot}5.$$

$$\text{,, Mode II} \ \ldots\ldots\ldots \quad 2\pi\frac{\mathfrak{T}}{T_2} = 111^\circ$$

$$\text{,, Mode III} \ \ldots\ldots\ldots \quad 2\pi\frac{\mathfrak{T}}{T_3} = 250^\circ.$$

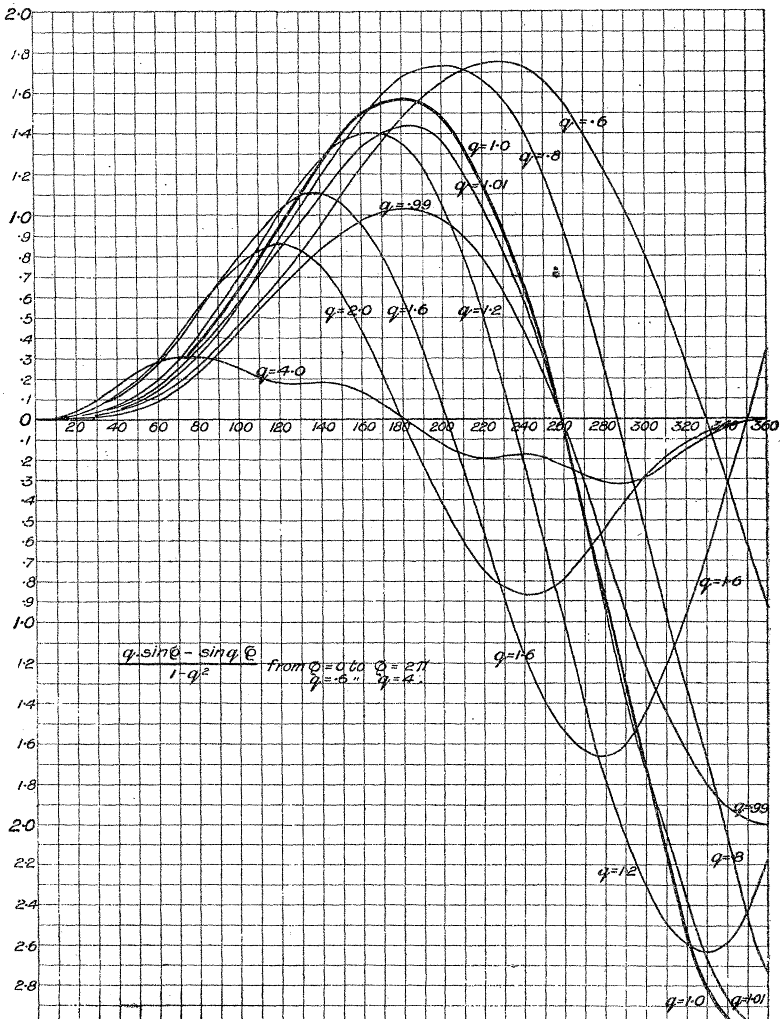
If then Diagram 11 had curves for all values on it, we should, in order to determine the deflection (due to vibration evoked in the mth mode by the nth term of the harmonic series) of the muzzle as the shot leaves it, merely have to take the ordinate of the curve for which $q = q_{nm}$ at the abscissa $2\pi\frac{\mathfrak{T}}{T_m}$, and multiply this ordinate by $\theta_m p d A_n$ as given in Table I, but the diagram, to avoid confusion, has shown on it only curves relating to a few values of q.

Using, however, the values of q_{nm} given in Table IV, and computing θ_{nm} for these values, by 24 it is found that

Table V.

$\theta_{11} = 19'{\cdot}5$ up.	$\theta_{12} = 28'{\cdot}5$ down.	$\theta_{13} = 60'$ up.
$\theta_{21} = 4'{\cdot}7$ up.	$\theta_{22} = 4'{\cdot}8$ down.	$\theta_{23} = 25'$ down.
$\theta_{31} = 1'{\cdot}55$ up.	$\theta_{32} = 2'{\cdot}05$ down.	$\theta_{33} = 4'{\cdot}9$ down.

Diagram 11.



Hence, adding these results, we find for the total upward deflection of $85'\cdot85$, a downward deflection of $65'\cdot25$, or finally, a resultant of $20'\cdot6$, as the angle which the instantaneous axis makes in an upward direction with the unstrained axis of the barrel, at the moment of the shot leaving the muzzle.

The course of the shot differs from instantaneous axis of the barrel by an amount depending on the ratio of the transverse linear velocity of the muzzle (due to the vibration) to the muzzle velocity of the shot. The transverse velocity v' of the muzzle consequent on the nth term vibration in the mth mode, can be obtained by differentiating θ_{nm} with respect to t, and multiplying by R_m (the distance of the nearest node of the mth mode from the muzzle). We then find the ratio v'/v

$$= \frac{2\pi R_m \theta_m p d A_n q_{nm}}{V_m T_m (1 - q^2_{nm})} \left(\cos 2\pi \frac{\mathfrak{T}}{T_m} - \cos 2\pi \, q_{nm} \frac{\mathfrak{T}}{T_m} \right) \ldots\ldots \quad (25).^*$$

Computing from this a table of corrections of angle corresponding to Table V representing the alterations of the values of the angles in Table V depending on the vertical linear speed of the barrel, we have approximately

Table VI.

	I.	II.	III.
1.	$4'\cdot6$ up.	$2'\cdot5$ down.	$5'\cdot0$ up.
2.	$0'\cdot35$	$0'\cdot0$	$0'\cdot0$
3.	$0'\cdot21$	$0'\cdot8$	$0'\cdot0$

or on the whole $6'\cdot9$ of upward inclination must be added to the $20'\cdot6$ found from Table V, so that the flight of shot lies $27'$ nearly above the direction of the unstrained axis.

The actual jump found by experiment for the Lee-Enfield rifle is, I believe, nearly about this amount, but from the uncertainty of the positions found for the nodes in the neighbourhood of the breech, and the small number of terms computed, as well as the doubtful approximation to the pressure curve, no great accuracy could be expected. The example is useful, however, and is introduced to show that the jump depends on the difference between comparatively large quantities, many of which are sure to be varying rapidly with q_{nm}.

The variations of q_{nm} may be caused either by the variation of T_m or t_n. For each individual rifle T_m of course is constant, depending as

* It may be noticed that in (24) and (25) $\sin 2\pi \, q_{nm} \frac{\mathfrak{T}}{T_m}$ must $= 0$, and $\cos 2\pi \, q_{nm} \frac{\mathfrak{T}}{T_m} = \pm 1$.

it does only on the elasticity and mass of the weapon, but t_n and A_n depend on the rapidity and rate of the explosion.

Suppose that in place of assumed explosive a slower burning explosive were used, with a charge sufficient to give the same muzzle velocity. This would cause an increase in $\mathfrak{T}$ and t_n; that is, q_{nm} would be diminished, and, owing to the greater terminal pressure (see (15) *et seq.*) all the values of A_n for n odd would be increased in relative importance compared with those for n even. The result in the case of a small variation of this kind in the Lee-Enfield would be an increased upward jump.

A lower muzzle velocity would also correspond to an increase of $\mathfrak{T}$, and would give an increased upward jump in this rifle, and at some particular range it should be found that the variation of jump and variation of initial velocity compensate one another, and that for moderate variations of charge the sighting at this range does not require alteration.

The natural periods of the rifle may be altered either by adding mass, or shortening the barrel. In the first case $\mathfrak{T}$ will remain unaltered, and q_{nm} will increase; thus the tendency of a small mass added near the muzzle will be to make the rifle shoot low.

If the barrel is shortened both T_m and $\mathfrak{T}$ are diminished, but the alteration in T_m (which depends on the square of length of the equivalent rod) is much more important than the alteration in $\mathfrak{T}$; hence a small shortening of the barrel may be expected to cause a considerable diminution in q_{nm} and a corresponding increase in upward jump.

The most important factors in these changes (as regards the Lee-Enfield) are $q_{1\cdot 2}$ and $q_{1\cdot 3}$, that is the effect of the first term of the harmonic expansion of the explosion curve in exciting the 2nd and 3rd mode vibration of the rifle.

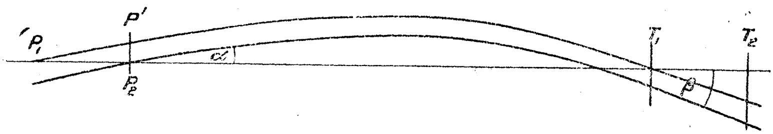
If ammunition could be made absolutely uniform in its action, "jump" would be of comparatively small importance, but the $\pm$ 40 feet per second by which the initial velocity of the service bullet varies may, by altering the factors on which "jump" depends, exaggerate with some classes of rifles, and diminish with others, the variation of the trajectory due to the effect of gravity and the altered initial velocity.

Suppose a rifle to be aimed and shot from P_1, fig. 12, so as to hit the centre of a target T_1 at range R, when the initial velocity is V. What will be the effect on the aim of a variation of the initial velocity?

Let α be the angle of elevation of the rifle and β the angle of descent of the bullet at T_1. Let P′ be the place in the trajectory of the shot (whose initial velocity is V) where the velocity has fallen to $V - v$. If a shot is fired from P_2 with the same sighting as was used

at P_1 and with the initial velocity $V - v$, the trajectory of this shot will always be a constant distance P_1P_2 below the trajectory through P_1, and will therefore strike the target T_1 at this distance below the centre. If a second target, T_2, is placed at a distance P_1P_2 ($= a$) behind T_1 so that $P_1T_1 = P_2T_2 = R$, the second target will be struck

FIG. 12.



$a\beta$ below the hit in the first target; hence since $P_1P_2 = a\alpha$, the error due to the variation of initial velocity is $a(\alpha + \beta)$. β may be found from the range tables of any rifle by the relation

$$\beta = R \,.\, da/dR.$$

Applying this to the Lee-Enfield, the following table shows the errors due to a variation of 40 feet per second in the initial velocity, on the assumption that the direction of the shot is not affected by "jump."

Table VII.

a = 54 feet = distance from muzzle at which the speed has fallen 40 feet per second.

Range in yards.	α.	β.	$a(\alpha + \beta)$.	$= \frac{a}{R}(\alpha + \beta)$.
			feet.	
100	4′	48′	0·141	1′·6
500	31′	43′	1·17	2′·16
1000	88′	144′	3·8	4′·35
1500	177′	320′	7·8	6′·0
2000	305′	570′	13·8	7′·8
2500	477′	980′	23·0	10′·5

These errors are comparable with, but, especially at the longer ranges, greater than what the best shots are liable to in practice, so that with this particular rifle the compensating action of the variation of "jump" is a distinct advantage.*

For some time I was under the impression that the complete elimination of the effect of "jump" which could be effected by a recoiling barrel, such as has been used in some repeating rifles, would lead to

* The fact that in this rifle variation of "jump" had a corrective effect was noticed by the late Sir Henry Halford.

improved accuracy in shooting; but in view of the above results it would appear that this is not the case.*

The present inquiry shows that in the design of a rifle it is most important to consider the relations between the explosion force and the natural periods of the rifle, considered as an elastic structure, and that probably the compensating effect above mentioned might be made of more use than it is at present.

For this purpose the explosion curves for various classes of ammunition and the variations to which they are liable should be accurately known, and the proportions and length of the barrel, as well as the attachment of the barrel to the stock, should be so arranged with regard to the nodes of the system as to make variation of "jump" with the variation of initial velocity most nearly balance, within certain ranges, the alteration in the trajectory which gravity would otherwise effect in virtue of the altered initial velocity.

To show the sort of advantage which may be obtained by this means, we may, for example, suppose the rifle to be so constructed that for some particular class of ammunition the variation of "jump" due to a ± 40 f.s. of initial velocity causes downward or upward variation of 6′ in the initial direction of the shot. Then by subtracting 6′ from E in Table VII, and multiplying by R, we get the following results :—

Table VIII.

Error due to ± 40 f.s. in initial velocity.

	Error	
	Without jump.	With jump.
100 yards	± 0·14 feet.	∓ 0·38 feet.
500 ,,	1·17 ,,	1·70 ,,
1000 ,,	3·8 ,,	1·26 ,,
1500 ,,	7·8 ,,	0·00 ,,
2000 ,,	13·8 ,,	± 3·15 ,,
2500 ,,	23·0 ,,	9·8 ,,

Such a correction, if it can be realised without an inconvenient construction of the mechanism, would be valuable for military purposes now that long-range fire is becoming of such great importance.

* There is another form of "jump," however, in the Lee-Enfield rifle, whose absence is most desirable, as it introduces horizontal movements of the barrel. It depends, not on the acceleration of the shot, but on the statical pressure of the powder gas acting on an unsymmetrical breech-closing action, and the remedy, as well as the disadvantages, are so clear in this case as not to call for further remark.